Belgium Algorithm Contest

Belgium Algorithm Contest
Round 3 - 2018

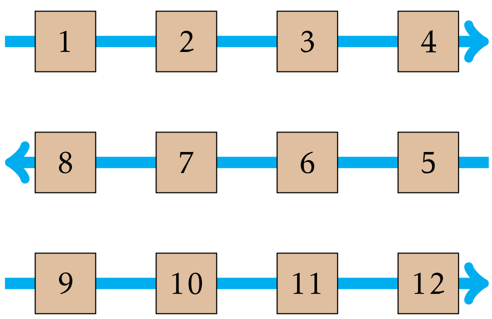
Do not open before the start of the contest.

This page is not really blank.

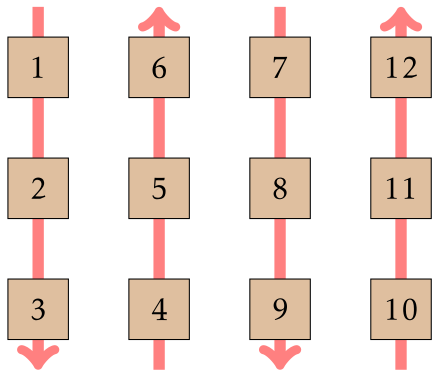
Belgium Algorithm Contest

Problem A
Easter eggs
Time limit: 1s

Alice and Bob are hunting for easter eggs. The eggs are hidden under some boxes that are disposed in an n by m grid. Alice will search the boxes horizontally and Bob will search them vertically. They both start from the first box at $(0,0)$ a go back and forth as illustrated in the following figure (the numbers indicate the order in which the boxes are searched).



Alice visit order



Bob visit order

The first to reach each egg will take it. If they both reach it at the same time, no one takes it.

Given the positions of the eggs, who will get more eggs?

Input

The first line of the input contains two integers r and c separated by a single space giving the number of rows and columns, respectively.

The second line contains a single integer k giving the number of eggs.

Then follow k lines each with two integers i and j giving the row and column of each of the eggs.

The boxes are numbered from top to bottom, left to right. So $(0,0)$ is the top left box and $(r-1,c-1)$ is the bottom right box.

Constraints

1. $1 \leq r, c, k \leq 25000$
2. $0 \leq i < r$
3. $0 \leq j < c$

Output

A single line with `alice` if Alice is the one to get more eggs, `bob` if it is Bob or `tie` if they both get the same amount.

Sample test cases

Input 1

```
3 4
2
2 1
2 3
```

Output 1

```
bob
```

Input 2

```
3 4
3
0 3
1 2
2 1
```

Output 2

```
alice
```

Input 3

```
4 4
4
0 3
1 2
2 1
3 0
```

Output 3

```
tie
```

Belgium Algorithm Contest

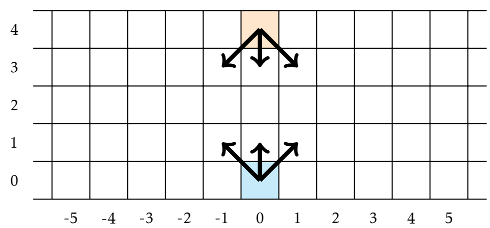
Problem B
Crossing paths
Time limit: 1s

You probably have noticed that when people are walking on the street in opposite directions and their paths cross, strange things happen and both people stumble without knowing what to do.

In this problem we will study the probability that they bump into each other if they start walking randomly. We model this by having two persons are on a grid of height d and infinite width. One is at position $(0,0)$ going up and the other at $(0,d-1)$ going down. At each step, the two persons perform a random step as follows:

- If the person going down is at position (x,y), it will go to either $(x-1,y-1),(x,y-1)$ or $(x+1,y-1)$ each with probably $1/3$. This person will stop upon reaching the bottom row ($y=0$).
- If the person going up is at position (x,y), it will go to either $(x-1,y+1),(x,y+1)$ or $(x+1,y+1)$ each with probably $1/3$. This person will stop upon reaching the top row ($y=d-1$).

The following figure illustrates the possible first moves when $d=5$.



The two persons move discretely one cell at a time and will cross paths if and only if either one of these two conditions are met:

- The two persons are on the same cell after the movement;
- The two persons switched cells after the movement. In other words, after the movement the person going up is where the person going down was before the movement and the person going down is where the person going up was.

What is the probability that the paths will cross?

Input

A single line with an integer d as described above.

Constraints

1. $3 \leq d \leq 32$.

Output

A single line with a number giving the probability that the two persons meet at some position. Your answer should have an absolute or relative error of at most 10^{-4}.

Sample test cases

Input 1	Output 1
3	0.3333333333333333

Input 2	Output 2
4	0.08641975308641975

Input 3	Output 3
7	0.1934156378600823

Belgium Algorithm Contest

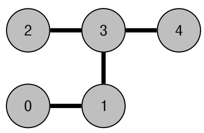
● Problem C
My dog ate my graph
Time limit: 7s

You little sister had a graph theory homework where she had to find an simple undirected graph with some very complicated properties. Unfortunately your dog Einie ate it. The good news is that she thinks that remembers the degree of each node of the graph.

Can you help her to rebuild the graph?

Example:

Suppose that $n = 5$ the degrees are $d_0 = 1, d_1 = 2, d_2 = 1, d_3 = 3$ and $d_4 = 1$. Then one possible answer is:



Input

The first line contains a single integer n giving the number of nodes of the graph.

The second line contains n integers $d_0, d_2, \ldots, d_{n-1}$ separated by single spaces such that d_i is the degree of node i.

Constraints

1. $1 \leq n \leq 10^5$
2. $0 \leq d_i \leq n - 1$
3. $\sum_{i=0}^{n} d_i \leq 2 \times 10^6$

Output

If there exists no simple undirected graph with nodes $0, 1, \ldots, n-1$ such that the degree of i is d_i then print `impossible`.

Otherwise print m lines, one for each undirected edge of the graph. Each line should contain two integers u and v meaning that there is an undirected edge between nodes u and v. Print only one edge for each pair of connected nodes. The order of the nodes does not matter.

It several answers exist, any one will be accepted.

Sample test cases

Input 1

```
5
1 2 1 3 1
```

Output 1

```
0 1
1 3
3 2
3 4
```

Input 2

```
5
2 4 2 2 3
```

Output 2

```
impossible
```

Belgium Algorithm Contest

Problem D
Can you program it?
Time limit: 6s

You just got a new robot in your contest goody bag and now want to program a robot to solve a maze. The robot accepts only four kinds of instructions.

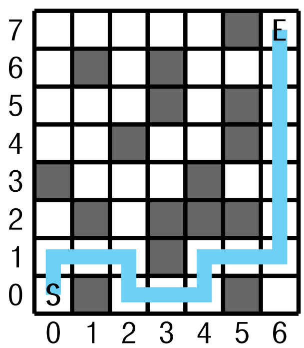
- `U-k` move up k steps;
- `D-k` move down k steps;
- `L-k` move left k steps;
- `R-k` move right k steps.

If an instruction makes the robot walk against a wall, the robot will break so you need to avoid this.

Since it has only a little amount of memory and the maze description already occupies most of it, you want to write the shortest possible program (minimum number of instructions) that makes the robot move from the source to the destination.

Example:

The following figure shows a maze and a possible shortest program to make the robot go the from start position S to the end position E.



shortest program

```
U-1
R-2
D-1
R-2
U-1
R-2
U-6
```

Input

The first line of the input contains two integers n and m giving the number of rows and columns of the maze, respectively.

Then follow n lines describing the maze. Each of those n lines contains a string of length m over the alphabet $\{., x, s, e\}$ representing each row of the maze. The x's represent the walls (that cannot be traversed by the robot), the .'s represent the empty spaces, s represents the starting position and e the ending position.

We guarantee that there exists a path from s to e.

Constraints

1. $2 \leq n, m \leq 1000$

Output

The first line should contain a single integer giving the number k of instructions in the smallest possible program.

Then must follow k lines each with an instruction as described above (see the samples for clarity).

If several solutions exists, any one will be accepted. Remember that you cannot make the robot walk into a wall.

Your output program can have at most 10^6 instructions.

Sample test cases

Input 1	Output 1

<pre>8 7
.....xe
.x.x...
...x.x.
..x..x.
x...x..
.x.xxx.
...x...
sx...x.</pre>
	<pre>7
	U-1
	R-2
	D-1
	R-2
	U-1
	R-2
	U-6</pre>

Input 2

```
7 5
....e
.x.x.
.....
x.x.x
.....
.x.x.
s....
```

Output 2

```
5
U-2
R-3
U-2
R-1
U-2
```

This page is not really blank.

Belgium Algorithm Contest

Problem E
How many primes?
Time limit: 5s

A prime number is a positive integer whose only positive divisors are 1 and itself.

The first few prime numbers are: $2, 3, 5, 7, 11, \ldots$. The k-th prime is denoted by p_k. For instance, $p_1 = 2, p_2 = 3$ and so on.

Given x, your task is to compute $\mathcal{P}(x)$, the number of primes less than or equal to x.

Input

A single integer x as described above.

Constraints

1. $1 \leq x \leq 10^{10}$

Output

A single line with the value of $\mathcal{P}(x)$, the number of primes that are smaller than or equal to x.

Sample test cases

Input 1	Output 1
5	3

Input 2	Output 2
100	25

Some theory:

To help you, you may be interested in the following.
Define

$$\phi(x,k) = \text{number of positive integers} \leq x \text{ will all prime factors} > p_k$$

$$P_r(x,k) = \text{number of positive integers} \leq x \text{ that are divisible by exactly } r \text{ (not necessarily distinct) primes all} > p_k$$

The following equalities hold:

$$\mathcal{P}(x) = \phi(x,\mathcal{P}(x^{1/3})) - P_2(x,\mathcal{P}(x^{1/3})) + \mathcal{P}(x^{1/3}) - 1 \tag{E.1}$$

$$\phi(x,k) = \phi(x,k-1) - \phi(x/p_k,k-1), \quad \text{for } k \geq 1 \tag{E.2}$$

Note: Feel free to not use these results in you solution.

This page is not really blank.